Set Theory

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1 Basic Info

Many of you have worked with sets in the past, or maybe you haven't? Anyway, let us begin by defining what a set really is. A set is nothing but a collection of things. Absolutely anything can be considered a set. Below you'll see just a sampling of items that could be considered as sets:

- Your Favorite Clothes
- A Coin Collection
- The Items In A Store
- The English Alphabet
- Even And Odd Numbers

Each entry in a set is known as an element. A set can have as many elements as you want it to have, which means that there could be no elements in a set or on the other hand, there also could be an infinite number of elements. Well, now that we have gotten over the basic stuff, let's move on!

2 Union, Intersection, and Subsets

In the previous section, we learned what an element and what a set was. So now we come to the question: What is a union? A union of two or more sets is another set that contains everything contained in the previous sets. In mathematical notation, the union is defined as $\cup$. Therefore, if A and B are sets, then $A \cup B$ represents the union of sets A and B. To see how this works in an actual problem, lets take a look at the example below:

Problem: We define set A to be the first five positive integers, and set B

to be the first five odd numbers starting with the number 5. List the elements in the union of sets A and B.

Solution: Set $A = \{1, 2, 3, 4, 5\}$ and Set $B = \{5, 7, 9, 11, 13\}$. Therefore, we are looking for the elements that are found in both sets. These elements are the numbers 1 through 5 and the numbers $5, 7, 9, 11, 13$. Thus, $A \cup B = \{1, 2, 3, 4, 5, 7, 9, 11, 13\}$. The number 5 does not need to be written twice, but all of the other elements needed to be listed.

Great job! We just finished learning what a union was, but we still have two other important things to go over. For simplicity's sake, let us assume we are at the mall and we walk into our favorite clothing store, and then we walk to another clothing store to see if they have anything different. However, we find out that both stores carry the same merchandise. This is an example of interesection. The intersection of two (or more) sets are those elements that they have in common, and is denoted as $\cap$. Therefore, if A and B are sets then the intersection (the elements they both have in common) would be $A \cap B$. There is one exception to finding an intersection of two sets - if the sets do not have anything in common, then we say that the intersection is the null (empty) set. This phoenomenon is denoted as $\emptyset$. Let's take a look at an example now:

Problem: We define the set $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 4, 5, 6\}$. Based on the elements provided, find $A \cap B$.

Solution: The elements they have in common are 3 and 5, and $A \cap B = \{3, 5\}$

Now that we learned what a set, element, intersection and union was, let's take a look at a subset. A subset is basically a set within a set. Not extremely complicated, but many problems deal with subsets because it is the most confusing part of set theory. The subset is defined to be $\subset$. Let us begin with definitions of four sets:

- Let A be the set of objects that you own in your home.
- Let B be the set of objects that you own, but which are kept on the second floor of your home.
- Let C be the set of objects that you own which are kept in your bedroom, which is also on the second floor
- Let D be the set of objects that you own which are kept in your bedroom nightstand.

Based on these definitions we can say D is contained within C, which in turn is contained within B, which in turn is contained within A. Or, in set theory notation, $D \subset C \subset B \subset A$! As usual, we need to be aware of exceptions. If even **one** element of one set is not contained within the other, then they are are not subsets. Let us take a look at two examples:

Problem: We define the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{2, 3, 4\}$. Based on the information, determine if the statement $B \subset A$ is true or not.

Solution: B is entirely within A (i.e. every element of B is also an element of A) so we can write $B \subset A$.

Now let us take a look where two sets do not form subsets:

Problem: We define the set $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4, 5, 6\}$. Based on the information, determine if the statement $B \subset A$ is true or not.

Solution: B would not be a subset of A since $6 \in B$, but 6 is not in A.

And that's all for this section!

3 Theorems

First of all, a theorem is a statement that can be proved. Below are a few theorems involving the concepts of union, intersection and subsets:

- If $A \subset B$ and $B \subset C$, then $A \subset C$.
- If $A \subset B$, then $A \cap B = A$.
- If $A \subset B$, then $A \cup B = B$.

  Due to space and time constraints, these theorems will not be proved in this packet, but they can be proved upon request.

4 Practice Problems

Congratulations! You have gotten through some of the tricker parts of Set Theory. I hope you have learned something new today, and hopefully you will be able to apply the techinques learned today during the competitions to come. Here are some practice problems that can be solved with important ideas in set theory:

1. Twenty-four dogs are in a kennel. Twelve of the dogs are black, six of the dogs have short tails, and fifteen of the dogs have long hair. There is only one dog that is black with a short tail and long hair. Two of the dogs are black with short tails and do not have long hair. Two of the dogs have short tails and long hair but are not black. If all of the dogs in the kennel have at least one of the mentioned characteristics, how many dogs are black with long hair but do not have short tails?

2. If $A = \{1, 3, 5\}$ and $B = \{0, 3, 6, 9\}$, find $[\{A \cap (B \cup A)\} \cap B] \cup A$.

3. Between 1933 and 1995 there were 11 presidents of the United States and 14 vice-presidents. If 9 of the vice-presidents were never president, how many of the presidents were never vice-president?

4. Participants in a two-day sports camp could register for only one of the days or for both of the days. There were 231 participants on Friday and 252 on Saturday. The total number of registered participants was 350. How many students attended the camp for both days?